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On the Existence of Spatially Uniform Scaling Laws in the Climate System

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Scale invariance (scaling) in a time series of an observable quantity is a symmetry law which when it exists can provide unique insights about the process in question. It describes variability and transitions at all scales and is often a result of nonlinear dynamics. It is well known that the spectra of atmospheric and climatic variables possess considerable power at low frequencies. Since "red" spectra often associate with scaling processes, it is reasonable to suppose that a search for scaling laws in climatic data might be fruitful. Consequently, the search for scaling in such data over the past decade has produced some exciting ways to describe climate variability. In the past and lately, there has been a growing interest in the existence of uniform in space temporal scaling laws for observable properties of the climate system, since such a property would provide a common rule describing temporal variability everywhere on the globe. Here we show that in spatially extended systems, uniform in space scaling demands that global averages be time invariant. A corollary to this is that where global averages do exhibit temporal variability, as in our climate system, spatial variation in scaling properties is required.

1 Introduction

A scaling (fractal) process y(t) satisfies the relationship $y(t) \stackrel{d}{=} \sigma^{-1} y(\lambda t)$ where $\stackrel{d}{=}$ indicates equality in distribution and σ, $\lambda > 0$. This relationship indicates that the statistical properties at time scale t are related to the statistical properties at time scale λt. Consequently, any moment of order k, μ'_k, satisfies the relation $\mu'_k(t) = \sigma^{-1} \mu'_k(\lambda t)$. It is easy to show that the power law $\mu'_k(t) = At^H$ with $H = \log\sigma / \log\lambda$ is a solution to the last equation (Triantafyllou *et al.*[1]).

Recently, new approaches based on the theory of random walks have been developed to elucidate scaling in time series (Tsonis and Elsner[2], Viswanathan *et al.*[3]). According to these approaches a time series x(t) representing some observable (temperature, pressure, etc.) is mapped onto a random walk whose net displacement, y(t), after t time steps is defined by the running sum $y(t) = \sum_{i=1}^{t} x(i)$. For any walk a suitable statistical quantity that characterizes the walk is the root mean square fluctuation about the average displacement $F^2(t) = \overline{[\Delta y(t)]^2} - \overline{[\Delta y(t)]}^{\,2}$, where $\Delta y(t) = y(t_0 + t) - y(t_0)$ and the bars indicate an average over all positions t_0 in the walk. The calculation of F(t) can distinguish three types of behavior: 1) uncorrelated time series described by $F(t) \propto t^H$ with H=0.5 as expected from the central limit theorem; 2) time series exhibiting positive long-range correlations described by $F(t) \propto t^H$ with $H > 0.5$; and 3) time series exhibiting negative long-range correlations described by $F(t) \propto t^H$ with H<0.5. Markov processes with local correlations extending up to some scale also give H=0.5 for sufficiently large t. It is well known (Feder[4]) that the correlation function C(t) of future increments, y(t), with past increments, y(-t), is given by $C(t) = 2(2^{2H-1} - 1)$. For H=0.5, we have C(t)=0 as expected, but for H≠0.5 we have C(t)≠0 independent of t. This indicates infinitely long correlations and leads to a scale-invariance (scaling) associated with positive long-range correlations for H>0.5 (i.e. an increasing trend in the past implies an increasing trend in the future) and to scaling associated with negative long-range correlations for H<0.5 (i.e. an increasing trend in the past implies a decreasing trend in the future). Random walks with H≠0.5 are referred to as fractional Brownian motions (fBms). In theory the exponent H is related to the spectra of the y(t) function via a relation of the form $S(f) \propto f^{-(2H+1)}$ and to the spectra of the x(t) function via a relation of the form $S(f) \propto f^{-2H+1}$ where f is the frequency.

2. Analysis and results

Let us assume that records of some meteorological variable exist at a sufficiently large number of stations (m) evenly distributed over the globe. Let us further assume that all those stations obey the spatially uniform scaling law $F(t) \sim t^H$ with the same exponent H. If $x_j(t)$, j=1,m are the records of the stations, then the global (planetary) mean of those records, $x_g(t)$, is given by:

$$x_g(t) = \frac{1}{m}\sum_{j=1}^{m} x_j(t) \tag{1}$$

It follows that the displacement of the random walk generated by the global mean record is:

$$y_g(t) = \sum_{i=1}^{t} x_g(i) = \sum_{i=1}^{t} \{\frac{1}{m}\sum_{j=1}^{m} x_j(i)\} = \frac{1}{m}\sum_{j=1}^{m} \left[\sum_{i=1}^{t} x_j(i)\right] \tag{2}$$

The sum in the bold brackets is the displacement of the walk for a particular station. Since we have assumed that at each station the records obey the law $F(t) \sim t^H$ with the same H, then the outer sum is zero (as it represents the average displacement after n time steps of many random walkers with the same exponent). In this case the above equation reduces to $y_g(t)=0$ which will indicate that the global mean $x_g(t)$ is also zero at any time.

This theoretically derived result can be verified by simple computer simulations. Consider m stations at which some variable y has been observed and that this quantity scales with the law $F(t) \sim t^H$ with H=0.7 at all stations. For illustrative purposes, we have generated such a function y(t) for m stations by inverting power spectra of the form $f^{-(2H+1)}$. The formula used to generate y(t) functions for $t=1, N$ is given by $y(t) = \sum_{k=1}^{N/2} [Ck^{-a}(\frac{2\pi}{N})^{1-a}]^{1/2} \cos(2\pi t k/N + \phi_k)$ where C is a constant, N is the sample size, ϕ_k are $N/2$ random phases uniformly distributed in $[0, 2\pi]$, and $a=2H+1$ (Osborne and Provenzale[5]; Tsonis[6]). Then using $y(t) = \sum_{i=1}^{t} x(i)$ the time series x(t) for each station was produced. From all the available stations we then estimated the global mean $x_g(t)$ for two sample sizes m (Figure 1). For m=10 the global mean fluctuates significantly above zero but as m increases the global mean tends at all times to zero. This result demonstrates the theoretical proof provided by equation (2) that a spatially uniform scaling law requires time invariance in the global mean. This behavior is manifestly counter to that of our climate system, which exhibits nonlinearities and variability at all time scales. Indeed, in a recent study it was shown that for the global temperature record the relation between $y_g(t)$ and t involves multiple temporal scaling regimes (Tsonis *et al.*[7]).

3. Discussion

To those familiar with the theory of random walks this result may not be surprising. Nevertheless, due to limitations in data and other shortcomings, applying these ideas to problems in physical sciences is often misguided and the wrong conclusions are drawn (Koscienly-Bunde *et al.*[8]). From the above, it follows that in spatially extended systems displaying variability at all time scales temporal scaling must vary in space. The spatial distribution of scaling must, in some way, reflect the dynamics of the system. For the climate system, spatial variation in scaling has been clearly demonstrated in a recent study of the 500 hPa height field, which is hydrostatically linked to the mean temperature of the lower troposphere (Tsonis *et al.*[9]). In this work, local scaling patterns were linked to specific properties of the atmospheric general circulation (baroclinic instability, storm tracks and persistence of circulation anomalies).

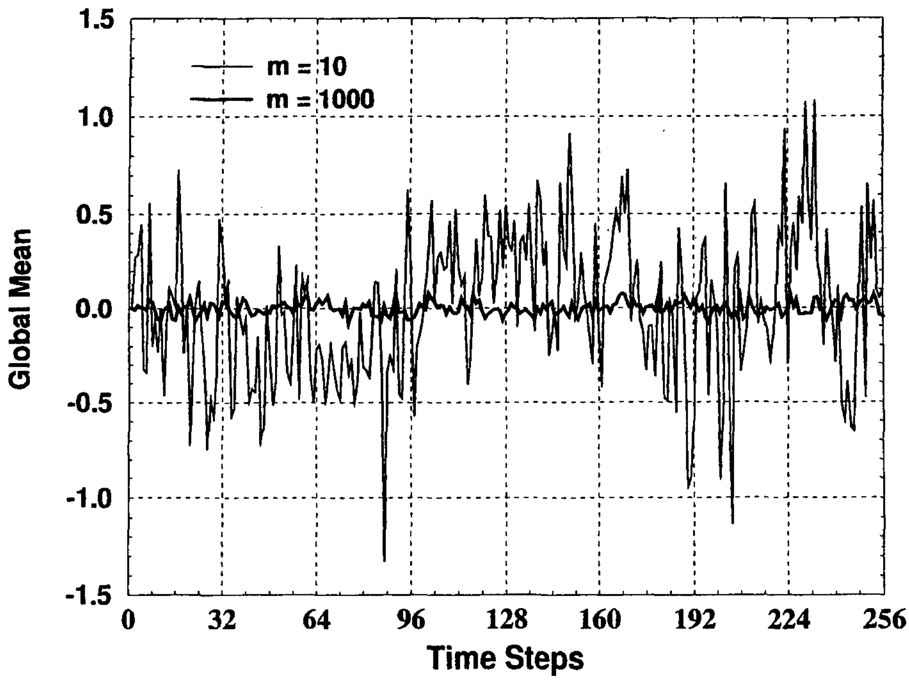

Figure 1. Simulated global mean value from m stations each one of them obeying the scaling law $F(t) \sim t^H$ with H=0.7 (see text for details).

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